

# Inverse Functions



1.) Find  $f(g(x))$  and  $g(f(x))$ .

$$f(x) = 2x + 3$$

$$g(x) = \frac{x-3}{2}$$

$$\begin{aligned}
 f(g(x)) &= 2 \left( \frac{x-3}{2} \right) + 3 \\
 &= \cancel{2} \left( \frac{x-3}{\cancel{2}} \right) + 3 \\
 &= x - 3 + 3 \\
 &= x
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= \frac{(2x+3)-3}{2} \\
 &= \frac{2x + \cancel{3} - 3}{2} \\
 &= \frac{\cancel{2}x}{2} \\
 &= x
 \end{aligned}$$

Functions are inverses of each other when...

$$f(g(x)) = x = g(f(x))$$

## Determining Whether the Inverse is a Function

**Horizontal Line Test**: The inverse of a relation is a function if and only if each horizontal line intersects the graph of the original relation in at most **one** point.

Find the inverse of the function, then determine whether the inverse is a function.

2.)  $k(x) = 3x - 5$

①  $y = 3x - 5$  ①\* Rewrite equation as  $y =$ .

②  $x = 3y - 5$  ②\* Switch  $x$  &  $y$ .

③  $x + 5 = 3y$   
 $\frac{x+5}{3} = y$

③\* Solve for  $y$ .

yes ✓  
 $\frac{x+5}{3} = k^{-1}(x)$

3.)  ~~$k(x) = \frac{x}{x-5}$~~

Is the inverse a function?

$y = \frac{x}{x-5}$

~~$x = \frac{y}{y-5}$~~

$x(y-5) = y$   
 $xy - 5x = y$

$xy = y + 5x$   
 $-y -y$

$xy - y = 5x$

$\frac{y(x-1)}{x-1} = \frac{5x}{x-1}$

$y = \frac{5x}{x-1}$

$h^{-1}(x) = \frac{5x}{x-1}$  ✓  
 yes!

4.)  $d(x) = -\sqrt[3]{x-8} + 3$       Is the inverse a function?

$$y = -\sqrt[3]{x-8} + 3$$

$$x = -\sqrt[3]{y-8} + 3$$

$$\frac{x-3}{-1} = \frac{-\sqrt[3]{y-8}}{-1}$$

$$\rightarrow (-x+3)^3 = (\sqrt[3]{y-8})^3$$

$$(-x+3)^3 = y-8$$

$$(-x+3)^3 + 8 = d^{-1}(x)$$

5.)  $g(x) = \frac{3}{2}x - 6$

Is the inverse a function?

6.) Verify  $f(x) = 2x - 4$  and  $f^{-1}(x) = \frac{1}{2}x + 2$  are inverses of each other.

$$\begin{aligned}
 &= 2(\quad) - 4 \\
 &= 2\left(\frac{1}{2}x + 2\right) - 4 \\
 &= 1x + 4 - 4 \\
 &= x \checkmark
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2}(\quad) + 2 \\
 &= \frac{1}{2}(2x - 4) + 2 \\
 &= x - 2 + 2 \\
 &= x \checkmark
 \end{aligned}$$

7.) Verify  $f(x) = -3x + 6$  and  $f^{-1}(x) = -\frac{1}{3}x + 2$  are inverses of each other.

# Homework

pg. 126: 9-12, 13-21 odd, 27-31 odd, 34, 43, 44

**PLEASE BRING YOUR TEXTBOOKS TO  
CLASS TOMORROW**