Solving Trig Equations

Using Identities

IDENTITIES

Quotient Identities	
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities	
★ $\sin^2\theta + \cos^2\theta = 1$ ★	
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

Reciprocal Identities		
$\sin\theta = \dfrac{1}{\csc\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$
$\csc\theta = \dfrac{1}{\sin\theta}$	$\sec\theta = \dfrac{1}{\cos\theta}$	$\cot\theta = \dfrac{1}{\tan\theta}$

Tips For Using Identities To Solve Trig Equations

* If the equation has multiple trig functions, try to convert all of them to the same trig function.
* If one or more of the trig functions are squared, try using one of the Pythagorean Identities to rewrite the equation, then simplify to get a factorable equation.
* When all else fails, turn your equation into sines and cosines.

Solve over the interval $[0°, 360°)$

1.) $1+\sin x = 2\cos^2 x$

$\cancel{\sin^2 x} + \cos^2 x = 1$
$-\cancel{\sin^2 x}$ $\quad -\sin^2$
$\cos^2 x = 1 - \sin^2 x$

$0 = 2\cos^2 x - \sin x - 1$
$0 = 2(1-\sin^2 x) - \sin x - 1$

$0 = 2 - 2\sin^2 x - \sin x - 1$
$0 = -2\sin^2 x - \sin x + 1$

-2
$-2 \quad 1$
-1

$0 = (-2\sin^2 x - 2\sin x) + (1\sin x + 1)$
$0 = -2\sin x(\sin x + 1) + 1(\sin x + 1)$
$0 = (-2\sin x + 1)(\sin x + 1)$

$-2\sin x + 1 = 0$
$\sin x = 1/2$
$x = 30°, 150°$

$\sin x = -1$
$x = 270°$

Solve over the interval $[0, 2\pi)$

2.) $\csc^2 x + 2\cot^2 x = 4$

$\csc^2 x + 2\cot^2 x - 4 = 0$

$\frac{1}{\sin^2 x} + \frac{2\cos^2 x}{\sin^2 x} - 4 = 0$

$\frac{1 + 2\cos^2 x}{\sin^2 x} = 4$

$\frac{1 + 2(1 - \sin^2 x)}{\sin^2 x} = 4$

$\frac{1 + 2 - 2\sin^2 x}{\sin^2 x} = 4$

$\cancel{\frac{3 - 2\sin^2 x}{\sin^2 x} = \frac{4}{1}}$

$4\sin^2 x = 3 - 2\sin^2 x$

$6\sin^2 x = 3$

$\sin^2 x = 1/2$

$\sin x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\csc^2 x + 2\cot^2 x = 4$

$(1 + \cot^2 x) + 2\cot^2 x = 4$

$1 + 3\cot^2 x = 4$

$3\cot^2 x = 3$

$\cot^2 x = 1$

$\cot x = \pm 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solve over the interval $[0, 2\pi)$

3.) $\sec x - 2\tan x = 0$

$\frac{1}{\cos x} - \frac{2\sin x}{\cos x} = 0$

$\frac{\cancel{\cos x} \cdot (1 - 2\sin x)}{\cancel{\cos x}} = 0 \cdot \cos x$

$1 - 2\sin x = 0$

$\sin x = 1/2$

$x = \pi/6, 5\pi/6$

Solve over the interval $[0°, 360°)$

4.) $\sec x - \tan x = \cos x$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} = \cos x$$

$$\frac{1-\sin x}{\cos x} = \cos x$$

$$1 - \sin x = \cos^2 x$$

$$1 - \sin x = 1 - \sin^2 x$$

$$\sin^2 x - \sin x = 0$$

$$\sin x(\sin x - 1) = 0$$

$\sin x = 0$ $\sin x = 1$

$$x = 0°, 180°, 90°$$