

Chapter 2 Solving polynomials
with imaginary solutions

Write the following in factored form then list all the zeros.

1.) $f(x) = x^4 - 4x^3 + 7x^2 - 12x + 12$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 7 & -12 & 12 \\ & \downarrow & 2 & -4 & 6 & -12 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$$

$$x^3 - 2x^2 + 3x - 6 = 0$$

$$x^2(x-2) + 3(x-2) = 0$$

$$(x^2 + 3)(x-2) = 0$$

$$x^2 = -3 \quad x = 2$$

$$x = \pm i\sqrt{3}$$

Factored Form: $f(x) = (x-2)(x-2)(x^2+3)$

Zero(s): $x = 2, 2, i\sqrt{3}, -i\sqrt{3}$

2.) $f(x) = 4x^4 + 26x^3 - 8x^2 + 39x - 21; -7$

$$\begin{array}{r|rrrrr} -7 & 4 & 26 & -8 & 39 & -21 \\ & \downarrow & -28 & 14 & -42 & 21 \\ \hline & 4 & -2 & 6 & -3 & 0 \end{array}$$

$$4x^3 - 2x^2 + 6x - 3 = 0$$

$$2x^2(2x-1) + 3(2x-1) = 0$$

$$(2x^2 + 3)(2x-1) = 0$$

$$2x^2 = -3 \quad 2x = 1$$

$$x^2 = -3/2$$

$$x = 1/2$$

$$x = \pm i\sqrt{3}/2$$

Factored Form: $f(x) = (x+7)(2x^2+3)(2x-1)$

Zero(s): $x = -7, 1/2, i\sqrt{3}/2, -i\sqrt{3}/2$

3.) $f(x) = x^3 - 10x^2 + 44x - 69$

$$\begin{array}{r|rrrr} 3 & 1 & -10 & 44 & -69 \\ & \downarrow & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$$x^2 - 7x + 23 = 0$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(23)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{-43}}{2} = \frac{7 \pm i\sqrt{43}}{2}$$

Factored Form: $f(x) = (x-3)(x^2-7x+23)$

Zero(s): $x = 3, \frac{7+i\sqrt{43}}{2}, \frac{7-i\sqrt{43}}{2}$

4.) $f(x) = x^4 - 4x^3 - 2x^2 - 12x - 15$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -2 & -12 & -15 \\ & \downarrow & 5 & 5 & 15 & 15 \\ \hline & 1 & 1 & 3 & 3 & 0 \end{array}$$

$$x^3 + x^2 + 3x + 3 = 0$$

$$x^2(x+1) + 3(x+1) = 0$$

$$x^2 + 3 = 0 \quad x + 1 = 0$$

$$x^2 = -3 \quad x = -1$$

$$x = \pm i\sqrt{3}$$

Factored Form: $f(x) = (x-5)(x+1)(x^2+3)$

Zero(s): $x = 5, -1, i\sqrt{3}, -i\sqrt{3}$

5.) $f(x) = x^5 + 3x^4 - 4x - 12$

$$\begin{array}{r|rrrrrr} -3 & 1 & 3 & 0 & 0 & -4 & -12 \\ & \downarrow & -3 & 0 & 0 & 0 & 12 \\ \hline & 1 & 0 & 0 & 0 & -4 & 0 \end{array}$$

$$x^4 + 0x^3 + 0x^2 + 0x - 4 = 0$$

$$x^4 - 4 = 0$$

$$(x^2 - 2)(x^2 + 2) = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$x^2 = -2$$

$$x = \pm i\sqrt{2}$$

6.) $f(x) = 2x^3 - x^2 + 2x - 1; \frac{1}{2}$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & 2 & -1 \\ & \downarrow & 1 & 0 & 1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

$$2x^2 + 0x + 2 = 0$$

$$2x^2 + 2 = 0$$

$$2x^2 = -2$$

$$x^2 = -1$$

$$x = \pm i$$

$$\left. \begin{array}{l} x = \frac{1}{2} \\ 2x = 1 \\ 2x - 1 = 0 \end{array} \right\}$$

Factored Form: $f(x) = (x+3)(x^2-2)(x^2+2)$ Factored Form: $f(x) = (2x-1)(2x^2+2)$

Zero(s): $x = -3, \sqrt{2}, -\sqrt{2}, i\sqrt{2}, -i\sqrt{2}$ Zero(s): $x = \frac{1}{2}, i, -i$

7.) $f(x) = 2x^4 - 11x^3 + 11x^2 - 176x - 336$

$$\begin{array}{r|rrrrr} 7 & 2 & -11 & 11 & -176 & -336 \\ & \downarrow & 14 & 21 & 224 & 336 \\ \hline & 2 & 3 & 32 & 48 & 0 \end{array}$$

$$2x^3 + 3x^2 + 32x + 48 = 0$$

$$x^2(2x+3) + 16(2x+3) = 0$$

$$x^2 + 16 = 0$$

$$x^2 = -16$$

$$x = \pm 4i$$

$$2x + 3 = 0$$

$$2x = -3$$

$$x = -3/2$$

8.) $f(x) = x^5 - 4x^4 - 4x^3 + 16x^2 - 45x + 180$

check notes
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Factored Form: $f(x) = (x-7)(2x+3)(x^2+16)$

Factored Form: _____

Zero(s): $x = -3/2, 7, 4i, -4i$

Zero(s): _____