Please remember there are many ways!

(1) $\tan^2 x - \sin^2 x = \sin^2 x\,(\sec^2 x - 1)$

$= \sin^2 x \sec^2 x - \sin^2 x$

$= \dfrac{\sin^2 x}{1} \cdot \dfrac{1}{\cos^2 x} - \sin^2 x$

$= \dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$= \tan^2 x - \sin^2 x$

A

(1) $\tan^2 x - \sin^2 x = \sin^2 x\,(\sec^2 x - 1)$

$\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\sin^2 x}{1} =$

$\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\sin^2 x \cos^2 x}{\cos^2 x} =$

$\dfrac{\sin^2 x - \cos^2 x \sin^2 x}{\cos^2 x} =$

B

$\dfrac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x} =$

$\sin^2 x \cdot \dfrac{(1 - \cos^2 x)}{\cos^2 x} =$

$\sin^2 x\left(\dfrac{1}{\cos^2 x} - \dfrac{\cos^2 x}{\cos^2 x}\right) =$

$\sin^2 x\,(\sec^2 x - 1) =$

(2)

$$\frac{\sec x+1}{\tan x} = \frac{\tan x}{\sec x-1} \cdot \frac{(\sec x+1)}{(\sec x+1)}$$

$$= \frac{\tan x(\sec x+1)}{\sec^2 x+\sec x-\sec x-1}$$

$$= \frac{\tan x\sec x+\tan x}{\sec^2 x-1}$$

$$= \frac{\tan x\sec x+\tan x}{\tan^2 x}$$

$$= \frac{\tan x(\sec x+1)}{\tan^2 x}$$

$$= \frac{\sec x+1}{\tan^2 x} \checkmark$$

(3)

$$\frac{\sec x+3}{1+\sec x} = \frac{\sec^2 x+2\sec x-3}{\tan^2 x}$$

$$= \frac{(\sec x+3)(\sec x-1)}{\sec^2 x-1}$$

$$= \frac{(\sec x+3)(\sec x-1)}{(\sec x-1)(\sec x+1)}$$

$$= \frac{\sec x+3}{\sec x+1} \checkmark$$

(3)

$$\frac{\sec x+3}{1+\sec x} = \frac{\sec^2 x+2\sec x-3}{\tan^2 x}$$

$$\frac{(\sec x-1)(\sec x+3)}{(\sec x-1)(\sec x+1)} = \frac{\sec^2 x+2\sec x-3}{\tan^2 x}$$

$$\frac{\sec^2 x+3\sec x-1\sec x-3}{\sec^2 x+\sec x-\sec x-1} =$$

$$\frac{\sec^2 x+2\sec x-3}{\sec^2 x-1} =$$

$$\frac{\sec^2 x+2\sec x-3}{\tan^2 x} \checkmark$$

④ $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$

$\frac{\sec x}{1} \cdot \frac{1}{\cos x} - \frac{\tan x}{1} \cdot \frac{1}{\cot x} =$

$\frac{1}{\cos x} \cdot \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\tan x}{1} =$

$\frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\cos x} =$

$\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} =$

$\frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$ ✓

⑤

$\cos x \cot x = \frac{1-\sin^2 x}{\sin x}$

$\frac{\cos x}{1} \cdot \frac{\cos x}{\sin x} =$

$\frac{\cos^2 x}{\sin x} =$

$\frac{1-\sin^2 x}{\sin x} =$ ✓

$\cos x \cot x = \frac{1-\sin^2 x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$= \frac{\cos x \cdot \cos x}{\sin x}$

$= \cos x \cdot \frac{\cos x}{\sin x}$

✓ $= \cos x \cdot \cot x$

⑥ $\csc^2 x(\tan^2 x - \sin^2 x) = \tan^2 x$

$\csc^2 x \tan^2 x - \csc^2 x \sin^2 x =$

$\frac{1}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{\cos^2 x} - \frac{1}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{1} =$

$\frac{1}{\cos^2 x} - 1 =$

$\sec^2 x - 1 =$

$\tan^2 x =$ ✓

⑦ $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

$(\sin x + \cos x)(\sin x + \cos x) + (\sin x - \cos x)(\sin x - \cos x) =$

$\sin^2 x + \cancel{\sin x \cos x} + \cancel{\sin x \cos x} + \cos^2 x + \sin^2 x - \cancel{\sin x \cos x} - \cancel{\sin x \cos x} + \cos^2 x =$

* combine like terms

$2\sin^2 x + 2\cos^2 x =$

$2(\sin^2 x + \cos^2 x) =$

$2(1) = 2 =$ ✓

⑧ $\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x \cdot (1 + \cos x)}{1 - \cos x \cdot (1 + \cos x)}$

$= \frac{\csc x + \csc x \cos x}{1 - \cos^2 x}$

$= \frac{\csc x (1 + \cos x)}{\sin^2 x}$

$= \frac{1}{\sin x} \frac{(1 + \cos x)}{\sin^2 x}$

$= \frac{1 + \cos x}{\sin^3 x}$ ✓

(9.) $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) =$

$(\sin^2 x - \cos^2 x)(1) =$

$\sin^2 x - \cos^2 x =$

$\sin^2 x - (1 - \sin^2 x) =$

$\sin^2 x - 1 + \sin^2 x =$

$2\sin^2 x - 1 =$ ✓

(10.) $\dfrac{\sin x + 1}{\cos x} = \dfrac{\cos x}{1 - \sin x} \cdot \dfrac{(1 + \sin x)}{(1 + \sin x)}$

$\dfrac{\sin x + 1}{\cos x} = \dfrac{\cos x + \cos x \sin x}{1 - \sin^2 x}$

$= \dfrac{\cos x(1 + \sin x)}{\cos^2 x}$

$= \dfrac{1 + \sin x}{\cos x}$ ✓

(11.) $\dfrac{2 - \cos x}{1 + \cos x} = \dfrac{\cos^2 x - 3\cos x + 2}{\sin^2 x}$

$= \dfrac{(\cos x - 2)(\cos x - 1)}{1 - \cos^2 x}$

$= \dfrac{(\cos x - 2)(\cos x - 1)}{(1 - \cos x)(1 + \cos x)}$

$= \dfrac{(\cos x - 2)}{-1(1 + \cos x)}$

$= \dfrac{-\cos x + 2}{1 + \cos x} = \dfrac{2 - \cos x}{1 + \cos x}$ ✓

or?

$\dfrac{(1 - \cos x)}{(1 - \cos x)} \cdot \dfrac{2 - \cos x}{1 + \cos x} = \dfrac{\cos^2 x - 3\cos x + 2}{\sin^2 x}$

$\dfrac{2 - \cos x - 2\cos x + \cos^2 x}{1 - \cos^2 x} =$

$\dfrac{\cos^2 x - 3\cos x + 2}{\sin^2 x} =$ ✓

12.

$$\frac{1}{(1+\sin x)(1-\sin x)}^{(1+\sin x)} + \frac{1}{(1+\sin x)(1-\sin x)}^{(1-\sin x)} = 2\sec^2 x$$

$$\frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x} =$$

$$\frac{1+\sin x + 1 - \sin x}{1-\sin^2 x} =$$

$$\frac{1+1}{\cos^2 x} =$$

$$\frac{2}{\cos^2 x} =$$

$$2 \cdot \frac{1}{\cos^2 x} = 2\sec^2 x = ✓$$

13.

$$1 + \cos x = \frac{\sin^2 x}{1-\cos x}$$

$$= \frac{(1-\cos^2 x)}{1-\cos x}$$

$$= \frac{\cancel{(1-\cos x)}(1+\cos x)}{\cancel{1-\cos x}}$$

$$= 1+\cos x \; ✓$$

$$1+\cos x = \frac{\sin^2 x\,(1+\cos x)}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin^2 x + \sin^2 x \cos x}{1-\cos^2 x}$$

$$= \frac{\sin^2 x + \sin^2 x \cos x}{\sin^2 x}$$

$$= \frac{\sin^2 x\,(1+\cos x)}{\sin^2 x}$$

$$= 1+\cos x \; ✓$$

14.

$$\tan x(\cos x + \cot x) = \sin x + 1$$

$$\tan x \cos x + \tan x \cot x =$$

$$\frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1} + \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} =$$

$$\sin x + 1 = ✓$$

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$$\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3+\sin x}{1-\sin x}$$

$$= \frac{3+\sin x}{1-\sin x}\,\frac{(1+\sin x)}{(1+\sin x)}$$

$$= \frac{3+3\sin x + 1\sin x + \sin^2 x}{1-\sin^2 x}$$

$$= \frac{\sin^2 x + 4\sin x + 3}{\cos^2 x}$$

$$\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3+\sin x}{1-\sin x}$$

$$\frac{(\sin x + 3)(\sin x + 1)}{1-\sin^2 x} =$$

$$\frac{(\sin x + 3)(\sin x \cancel{+1})}{(1-\sin x)(1+\cancel{\sin} x)} =$$

$$\frac{\sin x + 3}{1-\sin x} =$$

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$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \cot x - \tan x$$

$$\frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x} =$$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} =$$

$$\cot x - \tan x =$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \cot x - \tan x$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$= ~~\cos x \sin x~~$

$$\frac{\cos^2 x}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos x \sin x}$$