

Find the six trigonometric functions with the given information. (8 points each)

1.)  $(2, -\sqrt{5})$   
 $x \quad y$   
 $(2)^2 + (-\sqrt{5})^2 = r^2$   
 $4 + 5 = r^2$   
 $3 = r$

2.)  $\csc \theta = \frac{2}{1}$  and  $90^\circ < \theta < 180^\circ$   
 $x^2 + 1^2 = 2^2$   
 $x^2 + 1 = 4$   
 $x = -\sqrt{3}$

$\frac{1}{\sqrt{5}}$   
 $\frac{1}{\sqrt{5}}$   
 $\frac{1}{\sqrt{5}}$

$\sin \theta = \frac{-\sqrt{5}}{3}$	$\csc \theta = \frac{-3\sqrt{5}}{5}$
$\cos \theta = \frac{2}{3}$	$\sec \theta = \frac{3}{2}$
$\tan \theta = \frac{-\sqrt{5}}{2}$	$\cot \theta = \frac{-2\sqrt{5}}{5}$

$\sin \theta = \frac{1}{2}$	$\csc \theta = \frac{2}{1}$
$\cos \theta = \frac{-\sqrt{3}}{2}$	$\sec \theta = \frac{-2\sqrt{3}}{3}$
$\tan \theta = \frac{-\sqrt{3}}{3}$	$\cot \theta = -\sqrt{3}$

3.)  $(-2\sqrt{3}, -2)$   
 $x \quad y$   
 $(-2\sqrt{3})^2 + (-2)^2 = r^2$   
 $12 + 4 = r^2$   
 $4 = r$

4.)  $\sin \theta = -\frac{1}{7}$  and  $\frac{3\pi}{2} < \theta < 2\pi$   
 $x^2 + (-1)^2 = 7^2$   
 $x^2 + 1 = 49$   
 $x = \sqrt{48} < \frac{16}{3} = 4\sqrt{3}$

$\sin \theta = -\frac{1}{2}$	$\csc \theta = -2$
$\cos \theta = \frac{-\sqrt{3}}{2}$	$\sec \theta = \frac{-2\sqrt{3}}{3}$
$\tan \theta = \frac{\sqrt{3}}{3}$	$\cot \theta = \sqrt{3}$

$\sin \theta = -\frac{1}{7}$	$\csc \theta = -7$
$\cos \theta = \frac{4\sqrt{3}}{7}$	$\sec \theta = \frac{7\sqrt{3}}{12}$
$\tan \theta = \frac{-\sqrt{3}}{12}$	$\cot \theta = -4\sqrt{3}$

Find the reference angle, one positive coterminal angle, and one negative coterminal angle for the following angles. (3 points each)

5.)  $\frac{3\pi}{4}$

6.)  $30^\circ$

7.)  $-723^\circ$

Ref.  $\angle = \frac{\pi}{4}$

Ref.  $\angle = 30^\circ$

Ref.  $\angle = 3^\circ$

Pos.  $\angle = \frac{11\pi}{4}$

Pos.  $\angle = 390^\circ$

Pos.  $\angle = 357^\circ$

Neg.  $\angle = -\frac{5\pi}{4}$

Neg.  $\angle = -330^\circ$

Neg.  $\angle = -3^\circ, -363^\circ, -1083^\circ$   
(any)

Convert each degree measure to radians and vice versa. (1 point each)

8.)  $325^\circ \cdot \frac{\pi}{180^\circ}$

$= \frac{65\pi}{36}$

9.)  $9.1 \cdot \frac{180^\circ}{\pi}$

$= 521.39^\circ$

10.)  $\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi}$

$= 210^\circ$

11.)  $160^\circ \cdot \frac{\pi}{180}$

$= \frac{8\pi}{9}$

12.)  $\frac{\pi}{12} \cdot \frac{180^\circ}{\pi}$

$= 15^\circ$

13.)  $1.2 \cdot \frac{180^\circ}{\pi}$

$= 68.75^\circ$

Find all values for  $\theta$ , if  $\theta$  is in the interval  $[0^\circ, 360^\circ)$ . (2 points each)

14.)  $\cos \theta = -\frac{1}{2}$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$  or  $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

$\theta = 120^\circ, 240^\circ$

15.)  $\sin \theta = \frac{\sqrt{3}}{2}$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$  or  $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\theta = 60^\circ, 120^\circ$

16.)  $\cot \theta = -1$

$(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$  or  $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

$\theta = 135^\circ, 315^\circ$

Find all values for  $\theta$ , if  $\theta$  is in the interval  $[0, 2\pi)$ . (2 points each)

17.)  $\csc \theta = 1$   $(0, 1)$

$\theta = \frac{\pi}{2}$

18.)  $\cos \theta = -\frac{\sqrt{3}}{2}$

$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$  or  $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

19.)  $\tan \theta = 0$   $(1, 0)$  or  $(-1, 0)$

$\theta = 0, \pi$

Find the exact trigonometric value. No decimals. (2 points each)

20.)  $\cos 240^\circ$

$\boxed{-\frac{1}{2}}$

21.)  $\csc \frac{7\pi}{6}$

$\boxed{-2}$

22.)  $\sec 135^\circ$

$\boxed{-\sqrt{2}}$

23.)  $\sin \frac{\pi}{6}$

$\boxed{\frac{1}{2}}$

24.)  $\tan 0$

$\boxed{0}$

25.)  $\cot \frac{5\pi}{4}$

$\boxed{1}$

Identify the quadrant(s) for  $\theta$  satisfying the given conditions. (1 point each)

26.)  $\sin \theta > 0, \cot \theta < 0$

$\boxed{\text{II}}$

27.)  $\sec \theta < 0, \tan \theta < 0$

$\boxed{\text{II}}$

28.)  $\csc \theta < 0$

$\boxed{\text{III and IV}}$

29.)  $\cos \theta < 0, \sin \theta < 0$

$\boxed{\text{III}}$

Evaluate each inverse function. (1 point each)

30.)  $\tan \left[ \tan^{-1} \left( \frac{\sqrt{3}}{3} \right) \right]$

$\boxed{\frac{\sqrt{3}}{3}}$

31.)  $\arcsin(-1)$

$\boxed{\frac{\pi}{2}}$

32.)  $\sin \left[ \cos^{-1} \left( \frac{1}{3} \right) \right]$

$\frac{x}{r}$   
 $1^2 + y^2 = 3^2$   
 $1 + y^2 = 9$   
 $y = \pm 2\sqrt{2}$

$\boxed{\frac{2\sqrt{2}}{3}}$

33.)  $\csc \left[ \arctan \frac{\sqrt{3}}{5} \right]$

$\frac{y}{x}$   
 $(\sqrt{3})^2 + 5^2 = r^2$   
 $3 + 25 = r^2$   
 $\sqrt{28} = r$   
 $2\sqrt{7} = r$

$\boxed{\frac{2\sqrt{21}}{3}}$

34.)  $\sin^{-1} \left( -\frac{\sqrt{2}}{2} \right)$

$\boxed{-\frac{\pi}{4}}$   
 OR:  $\boxed{\frac{7\pi}{4}}$

35.)  $\cos \left[ \arccos \left( -\frac{1}{2} \right) \right]$

$\boxed{-\frac{1}{2}}$

Evaluate each inverse function. (1 point each)

36.)  $\sin^{-1}\left[\tan\left(\frac{\pi}{4}\right)\right]$

$\sin^{-1}(1)$

$\boxed{\pi/2}$

37.)  $\arcsin(-1)$

$\boxed{-\pi/2}$

or  $\boxed{3\pi/2}$

38.)  $\cos^{-1}\left[\sin\left(\frac{3\pi}{4}\right)\right]$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\pi/4}$

39.)  $\arccos[\tan(\pi)]$

$\arccos(0)$

$\boxed{\pi/2}$

40.)  $\sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$\sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\pi/6}$

41.)  $\tan^{-1}\left[\arctan\left(-\frac{3\pi}{2}\right)\right]$

$\boxed{-\frac{3\pi}{2}}$

Find the six trigonometric functions with the given information. (8 points each)

42.)  $(\sqrt{2}, -\sqrt{2})$   
 $\begin{matrix} x & y \end{matrix}$

$(\sqrt{2})^2 + (-\sqrt{2})^2 = r^2$   
 $2 + 2 = r^2$

$2 = r$

43.)  $\cot \theta = \frac{3}{4}$  and  $0 < \theta < \frac{\pi}{2}$

$4^2 + 3^2 = r^2$

$16 + 9 = r^2$

$5 = r$

$y/r$	$\sin \theta = \frac{-\sqrt{2}}{2}$	$\csc \theta = -\sqrt{2}$
$x/r$	$\cos \theta = \frac{\sqrt{2}}{2}$	$\sec \theta = \sqrt{2}$
$y/x$	$\tan \theta = -1$	$\cot \theta = -1$

$\sin \theta = \frac{4}{5}$	$\csc \theta = \frac{5}{4}$
$\cos \theta = \frac{3}{5}$	$\sec \theta = \frac{5}{3}$
$\tan \theta = \frac{4}{3}$	$\cot \theta = \frac{3}{4}$