

**MATRICES**

A Matrix is simplified version of working with equations with multiple variables.

If a car company is building cars and trucks they can use matrices to determine the number of parts they will need over a given span of time, producing a particular number of vehicles. If each car needs 4 wheels, 2 bench seats, and 1 gas tank. Each truck needs 6 wheels, 1 bench seat and 3 gas tanks. Then we can set-up a matrix where each row and column are for a given part of the equation.

$$c \begin{matrix} w & s & g \\ \begin{bmatrix} 4 & 2 & 1 \\ 6 & 1 & 3 \end{bmatrix} \end{matrix}, \text{ where the } c=\text{cars, } t=\text{trucks, } w=\text{wheels, } s=\text{seats, and } g=\text{gas tanks}$$

Using matrices we can solve for all kinds of situations. Matrices have their own specific rules for adding, subtracting, multiplying, and dividing.

The size (dimension) of a Matrix is # **Rows by # Columns**. (Rows go across, columns up and down)

EXAMPLE:  $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix}$   $\leftarrow$  row  
 $\uparrow$  Column  
 Matrix B is a 3 x 2 matrix.

An element of a Matrix is the value in a particular position.

EXAMPLE:  $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -4 & -5 \end{bmatrix}$   $b_{row,column}$   $b_{1,2} = 2$   $\rightarrow$  1<sup>st</sup> then  $\downarrow$  2<sup>nd</sup>

2 is the element in the 1<sup>st</sup> row and 2<sup>nd</sup> column

Use the matrices below to answer all questions.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ -1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix} \quad D = \begin{bmatrix} -2 & -2 \\ 7 & 9 \\ 3 & 6 \end{bmatrix}$$

$$E = [2 \quad -8 \quad 13 \quad 5] \quad F = \begin{bmatrix} 4 \\ 7 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 2 & -4 \\ 3 & 5 & -5 \\ 1 & 1 & 6 \end{bmatrix} \quad H = \begin{bmatrix} -4 & 2 & 1 & 0 \\ -2 & -1 & 4 & 1 \end{bmatrix}$$

List the dimensions for the specified matrix

- |      |              |      |              |
|------|--------------|------|--------------|
| 1. E | <u>1 x 4</u> | 4. H | <u>2 x 4</u> |
| 2. F | <u>2 x 1</u> | 5. A | <u>3 x 3</u> |
| 3. D | <u>3 x 2</u> | 6. B | <u>3 x 2</u> |

Identify the element in the specified locations, If possible.

5. Matrix D,  $d_{2,1}$

5. 7

6. Matrix A,  $a_{2,3}$

6. 5

7. Matrix H,  $h_{4,1}$

7. DNE  $\emptyset$

8. Matrix E,  $e_{1,3}$

8. 13

If the Matrices are set equal to each other, *each element must be the same.*

Solve for all variables

9. 
$$\begin{bmatrix} 4 & x \\ y+3 & -8 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & z-8 \end{bmatrix}$$

$x = 0$

$y + 3 = 12$   
 $y = 9$

$-8 = z - 8$   
 $0 = z$

10. 
$$\begin{bmatrix} 2a+1 & 16 \\ 7-b & 1 \end{bmatrix} = \begin{bmatrix} 17 & 16 \\ -15 & c+4 \end{bmatrix}$$

$2a + 1 = 17$   
 $2a = 16$   
 $a = 8$

$7 - b = -15$   
 $-b = -22$   
 $b = 22$

$1 = c + 4$   
 $-3 = c$

**ADDING, SUBTRACTING, AND SCALAR MULTIPLICATION**

When Adding and Subtracting Matrices, the matrices *must be the same exact size!*

**Adding-** make sure you add ALL elements in the 2<sup>nd</sup> matrix.

**Subtracting** – make sure you subtract ALL elements in the 2<sup>nd</sup> matrix.

**Scalar Multiplication** – make sure you distribute the multiplier to ALL elements in the matrix.

Add/sub elements in the same positions the same spots

EXAMPLES:  $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 3 & 5 \\ 2 & -3 & 0 \end{bmatrix}$   $C = \begin{bmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$  Use the following matrices for these examples:

**Work:**

1.  $A+C = \begin{bmatrix} 1+3 & 0+0 & -2+2 \\ 2+1 & 3+(-1) & 5+0 \\ 2+0 & -3+(-4) & 0+3 \end{bmatrix}$  2.  $A-C = \begin{bmatrix} 1-(-3) & 0-0 & -2-2 \\ 2-1 & 3-(-1) & 5-0 \\ 2-0 & -3-(-4) & 0-3 \end{bmatrix}$  3.  $4A = \begin{bmatrix} 4(1) & 4(0) & 4(-2) \\ 4(2) & 4(3) & 4(5) \\ 4(2) & 4(-3) & 4(0) \end{bmatrix}$

**Answer:**

1.  $A + C = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 2 & 5 \\ 2 & -7 & 3 \end{bmatrix}$  2.  $A - C = \begin{bmatrix} 4 & 0 & -4 \\ 1 & 4 & 5 \\ 2 & 1 & -3 \end{bmatrix}$  3.  $4A = \begin{bmatrix} 4 & 0 & -8 \\ 8 & 12 & 20 \\ 8 & -12 & 0 \end{bmatrix}$

Name: \_\_\_\_\_

## MATRICES ON THE CALCULATOR

### ADDITION, SUBTRACTION, AND SCALAR MULTIPLICATION:

You can enter and manipulate matrices with your graphing calculator (see directions below):

$$A = \begin{bmatrix} 2 & -1 \\ 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -3 \\ 1 & 4 \end{bmatrix} \quad [A] + [B] = \begin{bmatrix} 8 & -4 \\ 8 & 8 \end{bmatrix}$$

### Entering the Matrix into the Calculator

- Go into the Matrix Menu
  - $2^{\text{nd}}$  **MATRX** (the  $x^{-1}$  key)
    - $\blacktriangleright$  (right arrow) over to **EDIT**
- Select a Matrix
  - **ENTER** for 1:[A] (This is Matrix A) OR
  - Use  $\blacktriangledown$  (down arrow) to select another Matrix in the list and push **ENTER**
- Input the Dimension (size) of the Matrix (*Rows x Columns*)
  - # of Rows **ENTER**
  - # of Columns **ENTER**
- Input the elements (values) of the Matrix
  - Type in each # so that the matrix in the calculator looks *exactly* like the matrix on the paper.
  - Use the arrow keys to move within the Matrix.
- When the Matrix is complete
  - $2^{\text{nd}}$  **QUIT**

### To Perform Operations on Matrices (Addition, Subtraction, Scalar Multiplication)

- Enter all the matrices you need to perform the operation(s).
- Go into the Matrix Menu
  - $2^{\text{nd}}$  **MATRX** (the  $x^{-1}$  key)
    - Under **NAMES**, press **ENTER** for 1:[A] (This is Matrix A) OR
    - Use  $\blacktriangledown$  (down arrow) to select another Matrix in the list and push **ENTER**
- Name of the first Matrix will appear on the Home screen. Example: [A]
- Push the desired operation key: **+**, **-**, **x** Example: [A] +

- Go into the Matrix Menu
  - $2^{\text{nd}}$  **MATRIX** (the  $x^{-1}$  key)
  - Use  $\blacktriangledown$  to select the other Matrix in the list and push **ENTER**
- Name of second Matrix will appear on the Home screen. Example: [A] + [B]
- Push **ENTER** and the result will be displayed on the Home screen.

Use your calculator to perform the appropriate operation on the given matrices

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 6 & 3 & -3 \\ 0 & -3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 7 & 0 \\ -4 & 1 & 8 \\ -2 & -4 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 0 \\ -9 & 2 \\ 1 & 5 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ -9 & -1 \\ 4 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 0 & -7 \\ 6 & 4 & 3 \end{bmatrix}$$

1.  $B + A$

1. 
$$\begin{bmatrix} 3 & 7 & 4 \\ 2 & 4 & 5 \\ -2 & -7 & 5 \end{bmatrix}$$

2.  $D - C$

2. 
$$\begin{bmatrix} -4 & 1 \\ 0 & -3 \\ 3 & -2 \end{bmatrix}$$

3.  $3E$

3. 
$$\begin{bmatrix} 3 & 0 & 9 \\ 12 & 0 & -21 \\ 18 & 12 & 9 \end{bmatrix}$$

4.  $B + A - C$

4. not possible. C has different dimensions

5.  $4A - 3E + B$

5. 
$$\begin{bmatrix} 3 & 7 & 7 \\ 8 & 13 & 17 \\ -20 & -28 & 8 \end{bmatrix}$$

6.  $A - B + E$

6. 
$$\begin{bmatrix} 0 & -7 & 7 \\ 14 & 2 & -18 \\ 8 & 5 & 6 \end{bmatrix}$$

7.  $-3B - 6A$

7. 
$$\begin{bmatrix} -12 & -21 & -24 \\ -24 & -21 & -6 \\ 6 & 30 & -27 \end{bmatrix}$$

8.  $8C - 3C$

8. 
$$\begin{bmatrix} 20 & 0 \\ -45 & 10 \\ 5 & 25 \end{bmatrix}$$

9.  $E + 2A - 9B$

9. 
$$\begin{bmatrix} -15 & -63 & 11 \\ 52 & -3 & -85 \\ 24 & 34 & 2 \end{bmatrix}$$

10.  $6C + 4D - 2C$

10. 
$$\begin{bmatrix} 16 & 4 \\ -72 & 4 \\ 20 & 32 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -5 \\ -1 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 11 \\ -2 \\ 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 & -4 \\ 8 & 3 & -4 \\ -9 & 2 & 5 \end{bmatrix}$$

$$D = [4 \quad -1 \quad 6]$$

$$E = \begin{bmatrix} 6 & -8 \\ 0 & 3 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & 4 \\ -5 & 3 \\ 1 & 6 \end{bmatrix}$$

$$G = \begin{bmatrix} 10 & 1 & 0 \\ 2 & -4 & 5 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 8 \\ -4 \\ 5 \end{bmatrix}$$

$$J = \begin{bmatrix} 1 & -4 & 7 \\ 3 & 6 & 9 \\ -2 & 0 & 2 \end{bmatrix}$$

$$K = \begin{bmatrix} 8 & 4 & -3 \\ 7 & 1 & 0 \end{bmatrix}$$

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List the dimensions for each specific matrix.

11. A 2 × 2

12. B 3 × 1

13. I 3 × 1

14. D 1 × 3

15. C 3 × 3

16. J 3 × 3

17. K 2 × 3

18. F 3 × 2

If possible, identify the element in the specific locations of the matrix.

19. Matrix C,  $c_{2,3}$  -4

20. Matrix K,  $k_{1,3}$  -3

21. Matrix A,  $a_{2,2}$  7

22. Matrix I,  $i_{1,1}$  8

23. Matrix D,  $d_{1,3}$  6

24. Matrix J,  $j_{3,1}$  2

25. Matrix B,  $b_{3,1}$  5

26. Matrix G,  $g_{2,3}$  5