College Algebra

Second Semester Final Review

Name: _____

Date: _____ Period: _____

**Fill in the blank with the correct identity.**

1.) $\sin^2 x =$ 1 - $\cos^2 x$

2.) $\tan^2 x =$ $\sec^2 x$ - 1

3.) $\csc^2 x =$ 1 + $\cot^2 x$

4.) $\cos^2 x =$ 1 - $\sin^2 x$

5.) $\cot^2 x =$ $\csc^2 x$ - 1

6.) $\sec^2 x =$ 1 + $\tan^2 x$

7.) $\sin^2 x$ + $\cos^2 x$ = 1

8.) $\frac{1}{\csc x} =$ $\sin x$

9.) $\frac{\cos x}{\sin x} =$ $\cot x$

10.) $\frac{1}{\cos x} =$ $\sec x$

11.) $\frac{\sin x}{\cos x} =$ $\tan x$

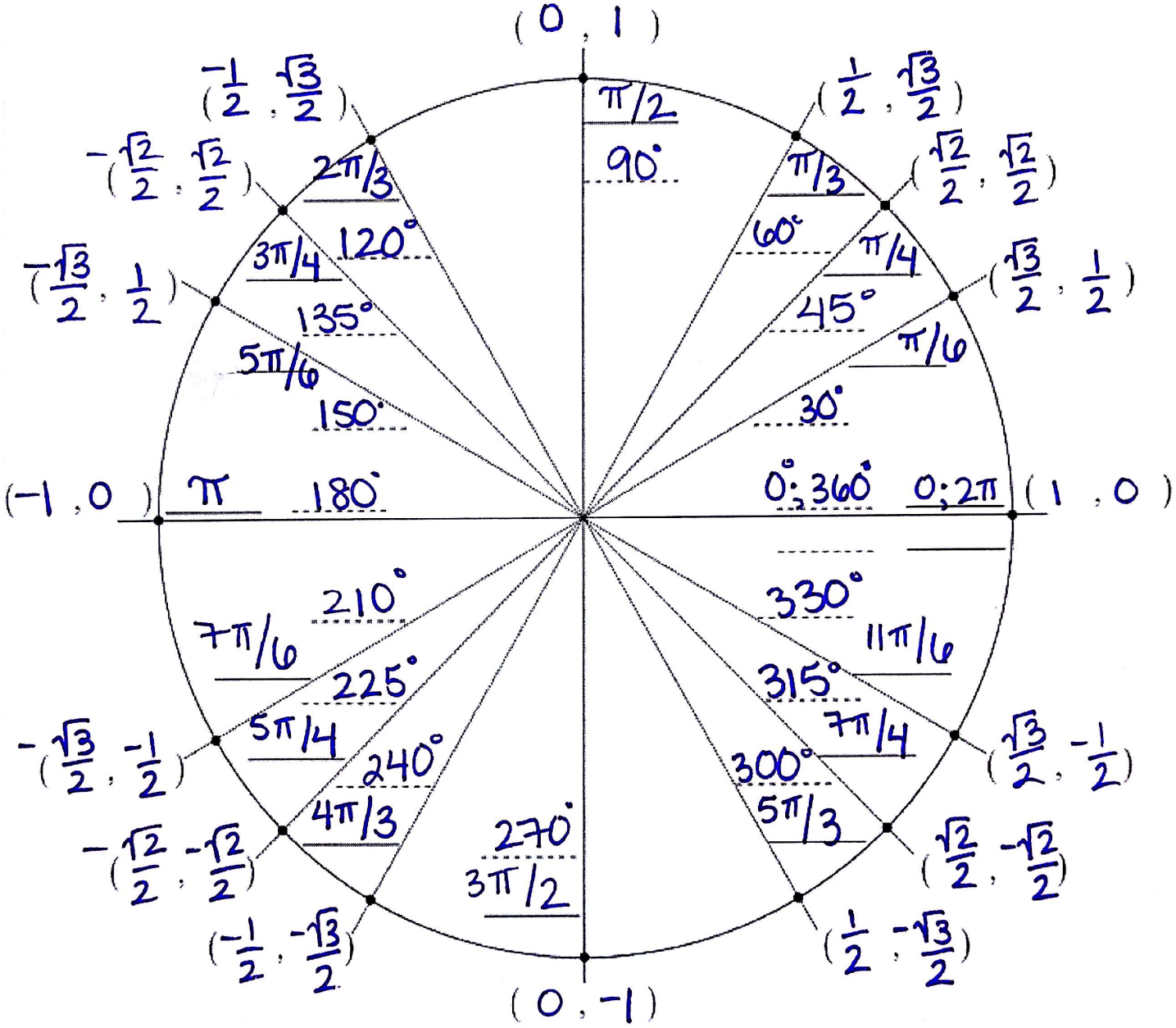
12.) $\frac{1}{\cot x} =$ $\tan x$

13.) $\frac{1}{\tan x} =$ $\cot x$

14.) $\frac{1}{\sec x} =$ $\cos x$

15.) $\frac{1}{\sin x} =$ $\csc x$

# Unit Circle


(0, 1)
π/2
90°
(1/2, √3/2)
π/3
60°
(√2/2, √2/2)
π/4
45°
(√3/2, 1/2)
π/6
30°
(1, 0)
0°; 360°
0; 2π
(−1/2, √3/2)
2π/3
120°
(−√2/2, √2/2)
3π/4
135°
(−√3/2, 1/2)
5π/6
150°
(−1, 0)
π
180°
210°
7π/6
225°
5π/4
(−√3/2, −1/2)
240°
4π/3
(−√2/2, −√2/2)
270°
3π/2
(−1/2, −√3/2)
(0, −1)
(1/2, −√3/2)
300°
5π/3
(√2/2, −√2/2)
315°
7π/4
(√3/2, −1/2)
330°
11π/6

**Solve each equation over the interval $[0°, 360°)$.**

16. $1 - 12cos^2x = -5$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = 45°, 135°, 225°, 315°}$

17. $(secx + \sqrt{2})(tanx + \sqrt{3}) = 0$

$\sec x = -\sqrt{2}$ $\quad$ $\tan x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{2}}{2}$

$\boxed{x = 135°, 225°, 120°, 300°}$

**Solve each equation over the interval $[0, 2\pi)$.**

18. $2sinxcosx + \sqrt{3}\sin x = 0$

$\sin x\,(2\cos x + \sqrt{3}) = 0$

$\sin x = 0$ $\quad$ $\cos x = -\sqrt{3}/2$

$\boxed{x = 0, \pi, 5\pi/6, 7\pi/6}$

19. $2cos^2x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = 1/2$ $\quad$ $\cos x = -2$

$\boxed{x = \pi/3, 5\pi/3}$

**Verify.**

20. $\frac{\sin^2 x}{\cos x} = \sec x - \cos x$

$\frac{1 - \cos^2 x}{\cos x} = \sec x - \cos x$

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sec x - \cos x$

$\sec x - \cos x = \sec x - \cos x$

✓

21. $\frac{\sin x - \sin^3 x}{\cos x} = \tan x \cos^2 x$

$\frac{\sin x\,(1 - \sin^2 x)}{\cos x} = \tan x \cos^2 x$

$\frac{\sin x\,(\cos^2 x)}{\cos x} = \tan x \cos^2 x$

$\frac{\sin x}{\cos x}\cos^2 x = \tan x \cos^2 x$

$\tan x \cos^2 x = \tan x \cos^2 x$

✓

**State the amplitude, period, and the horizontal and vertical shift for each function. Then graph.**

22. $y = -2\,cos(x - \pi) - 3$

Amplitude = 2

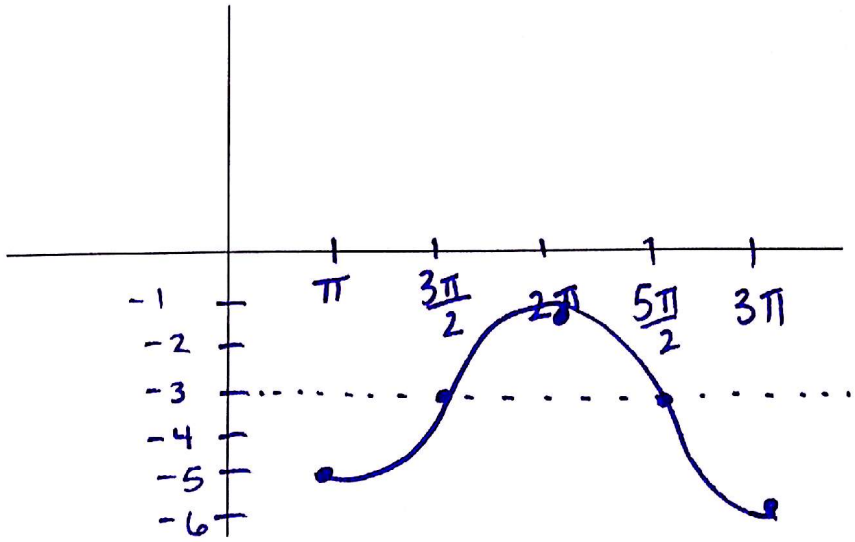
Period = $2\pi$

H. S. = R $\pi$

V. S. = D 3

D: $(-\infty, \infty)$

R: $[-5, -1]$



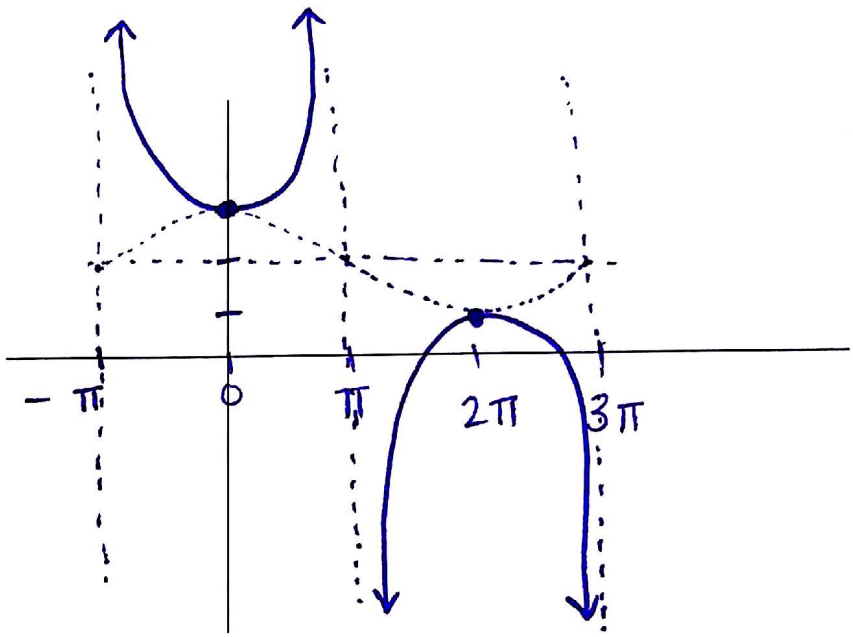

23. $y = csc\frac{1}{2}(x + \pi) + 2$

Amplitude = DNE

Period = $4\pi$

H. S. = L $\pi$

V. S. = U 2




24. $y = 4sin2x - 1$

Amplitude = 4

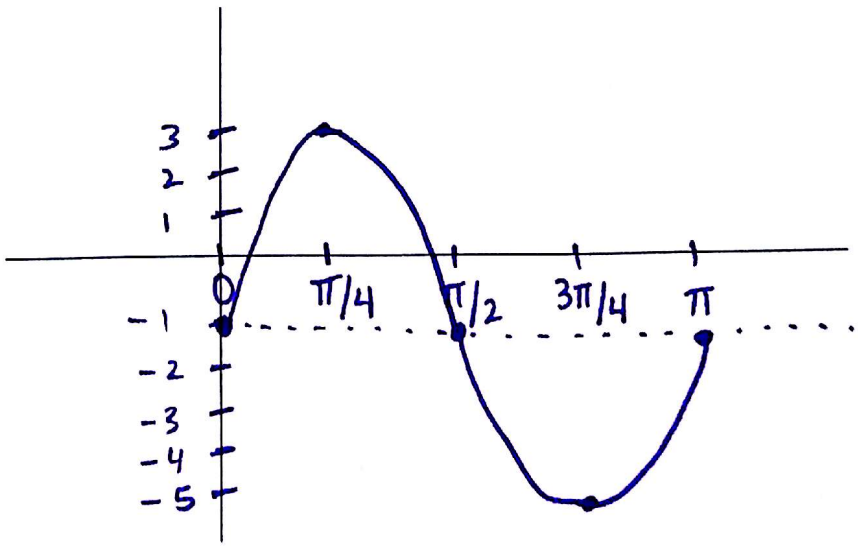
Period = $\pi$

H. S. = 0

V. S. = D 1

D: $(-\infty, \infty)$

R: $[-5, 3]$



**Solve the triangle. If necessary, round to the nearest hundredths.**

25. $A = 103.5^\circ \quad a = 41.5 \quad b = 27.2$

$$\frac{41.5}{\sin 103.5^\circ} = \frac{27.2}{\sin B}$$

$$41.5 \sin B = 27.2 \sin 103.5^\circ$$

$$B = \sin^{-1}\left(\frac{27.2 \sin 103.5^\circ}{41.5}\right)$$

$$B = 39.59^\circ \rightarrow 180^\circ - B = B'$$

$$\frac{41.5}{\sin 103.5^\circ} = \frac{c}{\sin 36.91^\circ}$$

| | |
|---|---|
| $A = 103.5^\circ$ $a = 41.5$ | ~~$A' = 103.5^\circ$ $a' = 41.5$~~ |
| $B = 39.59^\circ$ $b = 27.2$ | ~~$B' = 140.41^\circ$ $b' = 27.2$~~ |
| $C = 36.91^\circ$ $c = 25.63$ | ~~$C' = 63.91^\circ$ $c' =$~~ |

26. $A = 145^\circ \quad b = 4 \quad c = 14$

$$a^2 = 4^2 + 14^2 - 2(4)(14)\cos 145^\circ$$

$$a^2 = 303.745029$$

$$a = 17.43$$

$$\frac{17.43}{\sin 145^\circ} = \frac{4}{\sin B} \rightarrow B = 7.56^\circ$$

| | |
|---|---|
| $A = 145^\circ$ | $a = 17.43$ |
| $B = 7.56^\circ$ | $b = 4$ |
| $C = 27.44^\circ$ | $c = 14$ |

27. $a = 80.1 \quad b = 92.3 \quad c = 123.8$

$$80.1^2 = 92.3^2 + 123.8^2 - 2(92.3)(123.8)\cos A$$

$$-92.3^2 \quad -92.3^2 \quad -123.8^2$$

$$-123.8^2$$

$$\frac{-17429.72}{-22853.48} = \frac{-22853.48 \cos A}{-22853.48}$$

$$\cos^{-1}(\uparrow) = 40.3^\circ$$

$$\frac{80.1}{\sin 40.3^\circ} = \frac{92.3}{\sin B}$$

| | |
|---|---|
| $A = 40.3^\circ$ | $a = 80.1$ |
| $B = 48.19^\circ$ | $b = 92.3$ |
| $C = 91.51^\circ$ | $c = 123.8$ |

28. $A = 55.3^\circ \quad a = 22.8 \quad b = 24.9$

$$\frac{22.8}{\sin 55.3^\circ} = \frac{24.9}{\sin B}$$

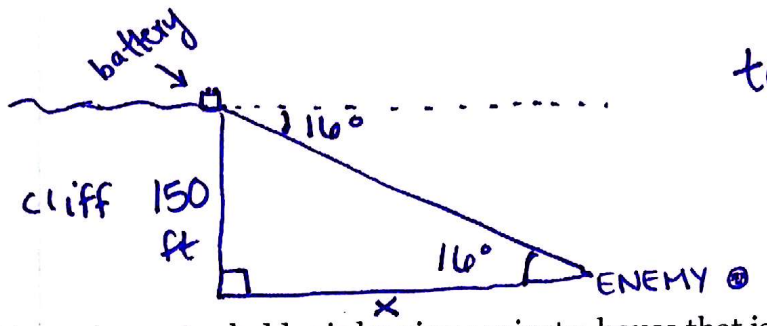
$$B = 63.88^\circ \rightarrow B' = 116.12^\circ$$

$$\frac{22.8}{\sin 55.3^\circ} = \frac{c}{\sin 60.82^\circ} \qquad \frac{22.8}{\sin 55.3^\circ} = \frac{c}{\sin 8.58}$$

| | |
|---|---|
| $A = 55.3^\circ$ $a = 22.8$ | $A = 55.3^\circ$ $a = 22.8$ |
| $B = 63.88^\circ$ $b = 24.9$ | $B = 116.12^\circ$ $b = 24.9$ |
| $C = 60.82^\circ$ $c = 24.21$ | $C = 8.58^\circ$ $c = 4.14$ |

**Solve. If necessary, round answers to the nearest hundredths.**

29. An artillery battery is at the top of a 150 foot cliff. If the enemy is located at an angle of depression of 16° from the top of the cliff, how far is the enemy from the base of the cliff?

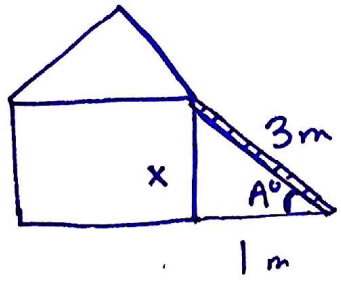



$\tan 16° = \frac{150}{x}$

$x = \frac{150}{\tan 16°}$

$\boxed{x = 523.11 \text{ ft}}$

30. A 3 meter ladder is leaning against a house that is 1 meter from the base of the house.
(a) What is the angle of elevation? (b) How far above the ground is the ladder touching the house?




$\cos A = \frac{1}{3}$

$\boxed{A = 70.53°}$

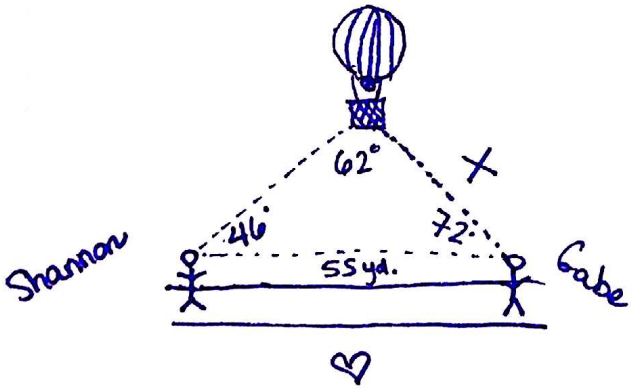
$1^2 + x^2 = 3^2$

$1 + x^2 = 9$

$x^2 = 8$

$\boxed{x = 2\sqrt{2} \text{ ft}}$ ← $\boxed{x = 2.83}$

31. Shannon and Gabe are walking toward each other on a path through the park. The path runs east-west. A hot air balloon is directly above the path between them. Shannon, sees the balloon when looking east at an angle of elevation of 46°. Gabe, sees the balloon looking west at an angle of elevation of 72°. If they are 55 yards apart, how far is the balloon from Gabe?

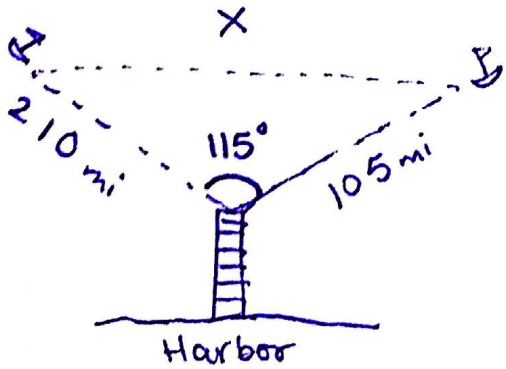



$\frac{55}{\sin 62°} = \frac{x}{\sin 46°}$

$\boxed{x = 44.8 \text{ yds.}}$

32. Two ships leave a harbor together, traveling on courses that have an angle of 115° between them. One ship traveled 105 miles, the other ship traveled 210 miles, how far apart are the ships?

* remember: we can only use pyth. thm. for ∟ Δs




$x^2 = 105^2 + 210^2 - 2(105)(210)\cos 115$

$\boxed{x = 271.59 \text{ mi}}$

**Find the following information for the given angle measure.**

33. $-671°$

Quadrant: I

Reference Angle: 49°

+ Co-terminal Angle = 49°

- Co-terminal Angle = −311°

34. $1274°$

Quadrant: III

Reference Angle: 14°

+ Co-terminal Angle = 914°

- Co-terminal Angle = −166°

**Convert to degrees.**

35. $\frac{6\pi}{5} \cdot \frac{180°}{\pi} = \boxed{216°}$

36. $\frac{2\pi}{9} \cdot \frac{180°}{\pi} = \boxed{40°}$

37. $-\frac{10\pi}{3} \cdot \frac{180°}{\pi} = \boxed{-600°}$

**Convert to radians.**

38. $342° \cdot \frac{\pi}{180°} = \boxed{\frac{19\pi}{10}}$

39. $-124° \cdot \frac{\pi}{180°} = \boxed{\frac{31\pi}{45}}$

40. $720° \cdot \frac{\pi}{180°} = \boxed{4\pi}$

**Given the following point on the terminal side of angle $\theta$, find the six trig functions for $\theta$.**

41. $(\overset{x}{-9}, \overset{y}{-40})$ $\quad (-9)^2 + (-40)^2 = r^2$ $\quad 41 = r$

$\frac{y}{r}$ $\sin\theta =$ −40/41 $\quad \csc\theta =$ 41/−40

$\frac{x}{r}$ $\cos\theta =$ −9/41 $\quad \sec\theta =$ 41/−9

$\frac{y}{x}$ $\tan\theta =$ 40/9 $\quad \cot\theta =$ 9/40

42. $(\overset{x}{7}, \overset{y}{-24})$ $\quad r = 25$

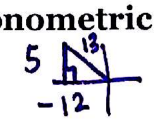
$\sin\theta =$ −24/25 $\quad \csc\theta =$ 25/−24

$\cos\theta =$ 7/25 $\quad \sec\theta =$ 25/7

$\tan\theta =$ −24/7 $\quad \cot\theta =$ 7/−24

**Find the values of the six trigonometric functions with the given information.**

43. $\sin\theta = \frac{\overset{y}{5}}{\underset{r}{13}}$ $\theta$ in quadrant II

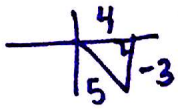



$\sin\theta =$ 5/13 $\quad \csc\theta =$ 13/5

$\cos\theta =$ −12/13 $\quad \sec\theta =$ 13/−12

$\tan\theta =$ 5/−12 $\quad \cot\theta =$ −12/5

44. $\tan\theta = -\frac{3}{4}$ $\quad \frac{3\pi}{2} < \theta < 2\pi$




$\sin\theta =$ −3/5 $\quad \csc\theta =$ 5/−3

$\cos\theta =$ 4/5 $\quad \sec\theta =$ 5/4

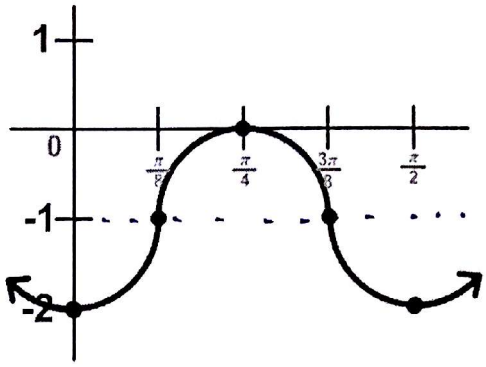
$\tan\theta =$ −3/4 $\quad \cot\theta =$ 4/−3

$\frac{8\pi}{3} = \frac{2\pi}{B} \rightarrow B = 3/4$

45. Write the equation of the sine function that has an amplitude of 3, period of $\frac{8\pi}{3}$, and is shifted up 5.

Equation: $y = 3\sin\frac{3}{4}(x) + 5$

46. Write the equation of the graph to the right.




Equation: $y = -\cos 4(x) - 1$

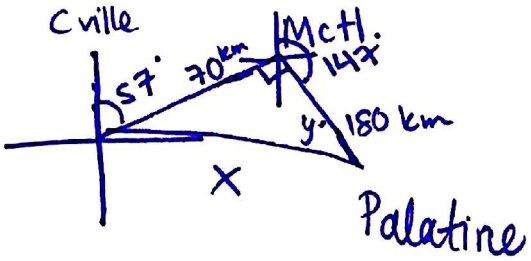
47. Find the period of the function $y = \tan\left(3x - \frac{\pi}{2}\right) + 5$

$y = \tan 3(x - \pi/6) + 5$

$\boxed{\pi/3}$

**Solve. If necessary, round answers to the nearest hundredths.**

48. The bearing from Carpentersville to McHenry is 57° and the bearing from McHenry to Palatine is 147°. If Carpentersville is 70 km from McHenry and McHenry is 180 km from Palatine, how far is Palatine from Carpentersville? What is the measure of the angle formed at Palatine?

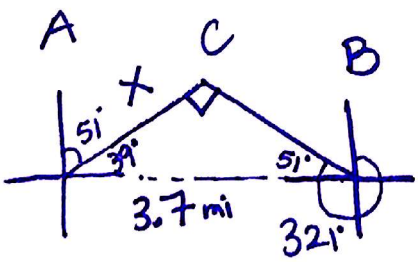



$70^2 + 180^2 = x^2$

$\boxed{193.13 \text{ km} = x}$

$\tan y° = \frac{70}{180} \rightarrow \boxed{y = 21.25°}$

49. Radar stations A and B are on an east-west line, 3.7 miles apart. Station A detects a plane at C, on a bearing of 51°. Station B detects the plane at a bearing of 321°. Find the distance from A to C.




$\sin 51° = \frac{x}{3.7}$

$\boxed{2.88 \text{ mi} = x}$

**Find the exact value for each of the following.**

50.) $\sin \frac{2\pi}{3}$

$\boxed{\frac{\sqrt{3}}{2}}$

51.) $\cos 90°$

$\boxed{0}$

52.) $\tan \frac{5\pi}{4}$

$\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{1}$

53.) $\csc \frac{\pi}{4}$

$\boxed{\sqrt{2}}$

54.) $\cot 330°$

$\boxed{-\sqrt{3}}$

55.) $\sec \frac{3\pi}{2}$

$\boxed{\text{undefined}}$

56.) $\cos \frac{5\pi}{3}$

$\boxed{\frac{1}{2}}$

57.) $\tan \pi$

$\boxed{0}$

58.) $\csc 150°$

$\boxed{2}$

59.) $\sin \frac{7\pi}{6}$

$\boxed{-\frac{1}{2}}$

60.) $\cot 0°$

$\boxed{\text{undefined}}$

61.) $\sec \frac{\pi}{3}$

$\boxed{2}$